An Ideal Six-Port Network Consisting of a Matched Reciprocal Lossless Five-Port and a Perfect Directional Coupler

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Abstract —A six-port consisting of a matched reciprocal lossless five-port in series with a perfect directional coupler is shown to have ideal six-port properties according to the theory developed by Engen. The q points are separated by 120° on a common circle. The matched five-port itself is shown to function as a four-way divider network and bears an interesting analogy with a matched nonreciprocal lossless three-port (circulator). This analogy is used to design several styles of matched symmetrical stripline five-ports for use in making six-port measurements.

I. Introduction

As an aid in determining what properties a six-port should have to enable accurate measurement of the reflection coefficient Γ to be made, Engen has introduced a useful diagram in the complex Γ-plane [1]. Engen has shown that, ideally, one power detector should be used to measure the power incident to the unknown load, and that the complex numbers q_1, q_2, and q_3 associated with the remaining three power detectors should be symmetrically distributed around the origin, i.e., separated by 120° at the vertices of an equilateral triangle, as in Fig. 1. A compact six-port network accomplishing these features consists of a matched, reciprocal, lossless five-port junction and a perfect directional coupler, as in Fig. 2.

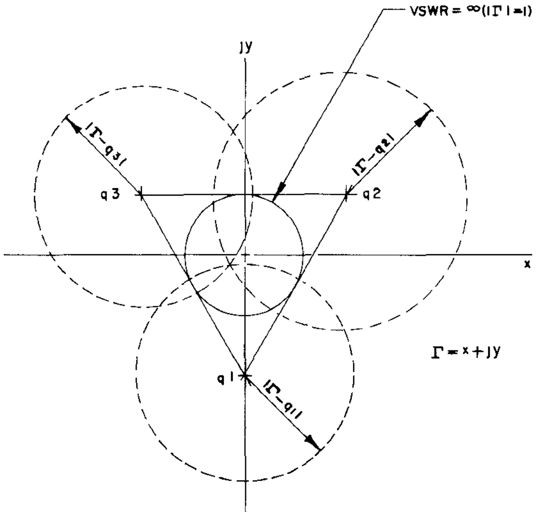

Fig. 1. Optimum location of q-points for six-port measurements. The reflection coefficient Γ is determined by the intersection of 3 circles whose radii are determined by power measurements and whose centers are q_1, q_2, and q_3.

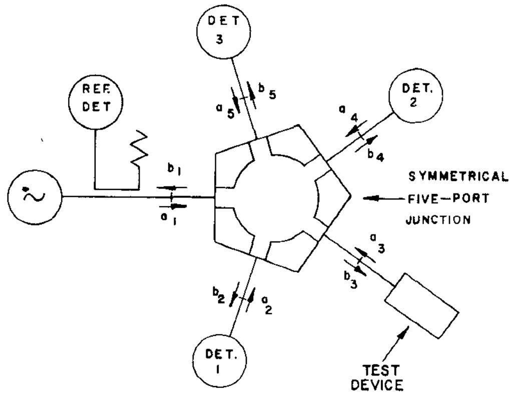

Fig. 2. Six-port configuration.

II. Properties of a Matched Reciprocal Symmetrical Lossless Five-Port

The linear scattering-matrix element–eigenvalue relations for symmetrical devices have often proven to be useful for determining the coupling properties of such matched networks. This is also the case for the five-fold symmetric reciprocal five-port. In this instance, the scattering-matrix element–eigenvalue relations have been given by Dicke [2] and are

$$S_{11} = (S_1 + 2S_2 + 2S_3)/5 \tag{1}$$

$$S_{12} = (S_1 + 2S_2\cos(2\pi/5) + 2S_3\cos(4\pi/5))/5 \tag{2}$$

$$S_{13} = (S_1 + 2S_2\cos(4\pi/5) + 2S_3\cos(2\pi/5))/5. \tag{3}$$

Because of symmetry, there will only be three independent S-matrix elements S_{11}, $S_{12}(=S_{15})$, and $S_{13}(=S_{14})$, and, likewise, three independent S-matrix eigenvalues S_1, S_2, and S_3. The eigenvalues S_2 and S_3 are doubly degenerate. The scattering-matrix eigenvalues represent the reflection coefficients of the three eigen-excitations of the junction, and consequently must have unit amplitude if the device is assumed to be lossless. Setting the phase of S_1 arbitrarily to 180°, we may write in the case of a lossless junction

$$S_1 = -1 \quad S_2 = e^{j\psi 2} \quad S_3 = e^{j\psi 3}. \tag{4}$$

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If, in addition, the device is matched

$$5S_{11} = \{-1+2e^{j\psi_2}+2e^{j\psi_3}\} = 0 \tag{5}$$

from (1). This condition can only be satisfied if $\psi_1 = -\psi_2 = \cos^{-1}(1/4) \simeq \pm 75.5°$. The scattering-matrix elements will now be determined to within a common phase factor from (2) and (3). We can summarize by saying that the conditions of losslessness and matching uniquely determine the coupling properties of a reciprocal symmetrical five-port.

Upon insertion of the above eigenvalues into (2) and (3), one finds that

$$|S_{12}| = |S_{13}| = 0.5 \tag{6}$$

$$\angle S_{13} = \angle S_{12} \pm 2\pi/3(120°). \tag{7}$$

The matched five-port thus functions as a four-way equal power divider in agreement with the results given by Dicke [2]. Consequently, if we assume the five-port of Fig. 2 to be matched, then a signal fed into the port attached to detector 1 will divide with equal amplitude and phase from the input and output ports attached to the generator and test device, respectively. Likewise, signals fed into the ports attached to detector 2 or detector 3 will divide with equal amplitude, but with a 120° phase difference from the input and output ports, respectively. It is readily shown that the q point distribution is that of an ideal six-port as given in Fig. 1 provided a perfect directional coupler is used with the reference detector [3]. It should be emphasized that ideal q point distribution will be obtained with any port of the five-port taken as the input port and any port as the output port. It is not necessary to use the configuration given in Fig. 2.

III. Analogy with Matched Nonreciprocal Lossless Three-Ports (Circulators)

It should be evident from the previous section that the problem of constructing an ideal six-port network has been reduced to the problem of matching a symmetrical five-port network. In this regard, an interesting analogy between the five-port junction and the symmetrical three-port circulator is helpful. It allows the theory that has been developed for matching three-port circulators to be applied to this new problem.

The linear S-matrix element–eigenvalue relations for the three-fold symmetric three-port circulator are [4]

$$3S_{11} = S_0 + S_1 + S_{-1} \tag{8}$$

$$3S_{12} = S_0 + S_1 e^{j2\pi/3} + S_{-1} e^{-j2\pi/3} \tag{9}$$

$$3S_{13} = S_0 + S_1 e^{-j\pi/3} + S_{-1} e^{j2\pi/3} \tag{10}$$

where, as in the case of the symmetrical reciprocal five-port, there will be three independent S-matrix elements S_{11}, S_{12}, and S_{13}, and three independent eigenvalues S_0, S_1, and S_{-1}. Assuming the three-port junction to be lossless implies that the S-matrix eigenvalues will be of unit amplitude so that again we may write

$$S_0 = -1 \quad S_1 = e^{j\psi_1} \quad S_{-1} = e^{-j\psi_{-1}} \tag{11}$$

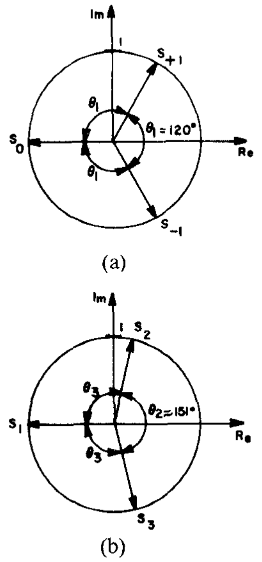


Fig. 3. Scattering matrix eigenvalue configurations for matched symmetrical lossless networks. (a) Nonreciprocal three-port, $\theta_1 = 2\cos^{-1}(0.5) = 120°$. (b) Reciprocal five-port, $\theta_2 = 2\cos^{-1}(0.25) \simeq 151°$.

where the phase of S_0 has arbitrarily been set to 180°. Assuming the junction to be matched ($S_{11} = 0$), it then follows from (8) that $\psi_1 = -\psi_{-1} = \pm 120°$. The relative phases of the eigenvalues for the matched nonreciprocal three-port and matched reciprocal five-port are given in Fig. 3. It now follows from (9) and (10) that either S_{12} or S_{13} will be of unit amplitude, i.e., perfect circulation will be obtained. The analogy between the reciprocal five-port junction and nonreciprocal three-port junction consists in the fact that in both cases the desired mode of operation, i.e., ideal six-port performance or perfect circulation can be obtained by matching the junction alone.

In fact, the analogy is even more remarkable than has been indicated so far. The results stated up until now have been for symmetrical devices. However, it is well known that any three-port lossless nonreciprocal device functions as a perfect circulator if it is matched at all ports. Results derived by Hieber and Vernon [5] can be used to show that a similar conclusion applies for a matched lossless reciprocal five-port even if it is not symmetrical. To do this, their theorem 2 and the phase relations of their Table I, which apply to nonsymmetrical five-ports, must be used.

IV. The Equivalent Admittance (Impedance) of the Symmetrical Five-Port

Because of the mathematical similarity between the nonreciprocal three-port and reciprocal five-port, the concept of an equivalent admittance can be introduced for a five-port in a manner very similar to the way it was introduced for the nonreciprocal three-port in [6]. If a two-port matching network is found which matches into this admittance, then the same matching network connected in each arm of the five-port will match the five-port, as in Fig. 4. In effect a multiport matching problem will be reduced to a much simpler one-port matching problem.

The derivation of expression for the real and imaginary parts G_e and B_e, respectively, of the equivalent admittance

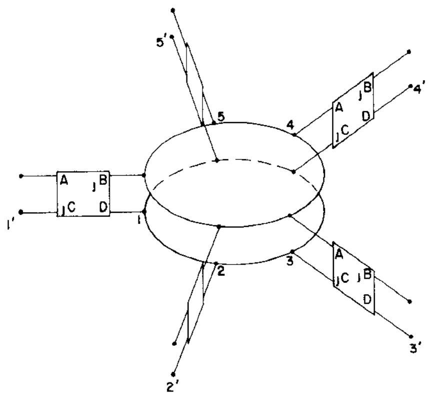


Fig. 4. A five-fold symmetric reciprocal five-port network with identical two-port matching networks connected in each arm. Unprimed quantities refer to those of the basic junction while primed quantities refer to those obtained after matching.

Y_e in terms of the junction eigensusceptances Y_1, Y_2, and Y_3, proceeds in a manner which is nearly identical to that given in [6] for the circulator and consequently will be given only in abbreviated form here. If Y_1, Y_2, and Y_3 are the eigensusceptances of the junction and Y_1', Y_2', and Y_3' the eigensusceptances of the symmetrical device which includes the matching networks, then

$$jY_i' = j\frac{C+DY_i}{A-BY_i}, \qquad i=1,2,3 \tag{12}$$

where A, B, C, and D are the elements of the $ABCD$ matrix of the matching network as in Fig. 4. From Section II we have that the conditions for a perfect match are

$$\psi_2' = \psi_1' \pm 104.5°, \quad \psi_3' = \psi_1' \mp 104.5° \tag{13}$$

Since $jY_i' = j\tan(-\psi_i'/2)$, the required conditions on the eigensusceptances can be obtained by taking the tangent of both sides of (13). These conditions are

$$-Y_2' = \frac{-Y_1' + \sqrt{5/3}}{1+\sqrt{5/3}\,Y_1'}, \quad -Y_3' = \frac{-Y_1' - \sqrt{5/3}}{1-\sqrt{5/3}\,Y_1'}. \tag{14}$$

Substituting (12) into (14) yields the following two equations after some algebra:

$$(B^2+D^2)\cdot G_e = 1 \tag{15}$$

$$(B^2+D^2)\cdot B_e = AB - CD \tag{16}$$

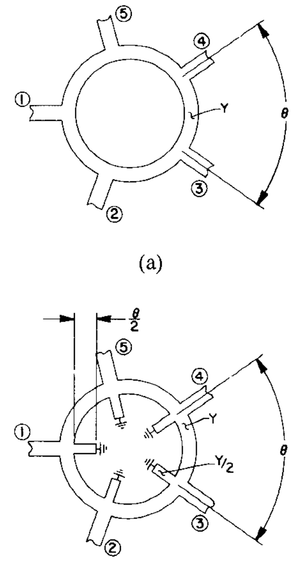
where G_e and B_e are the real and imaginary parts, respectively, of the equivalent admittance Y_{eq} and

$$Y_{eq} = \frac{(1+Y_1^2)G^* + j\left[G^{*2}Y_1 - (1-B^*Y_1)(B^*+Y_1)\right]}{G^* + (B^*+Y_1)^2} \tag{17}$$

$$G^* = \frac{1}{2}\sqrt{\frac{5}{3}}\left[\frac{Y_1Y_3+1}{Y_1-Y_3} - \frac{Y_1Y_2+1}{Y_1-Y_2}\right] \tag{18}$$

$$B^* = \frac{1}{2}\left[\frac{Y_1Y_3+1}{Y_1-Y_3} + \frac{Y_1Y_2+1}{Y_1-Y_2}\right]. \tag{19}$$

We have identified G_e and B_e as the real and imaginary parts of the equivalent admittance since (15) and (16) have the same form as the conditions for matching into a complex admittance [6]. The multiport matching problem has thus been reduced to a simpler one-port matching problem.



(a)

(b)

Fig. 5. Ring style five-ports with five-fold symmetry. Y is the admittance level of the lines connecting the various ports and θ their electrical length.

V. Matched Ring Style Five-Port

One of the simplest symmetrical five-ports to consider is the ring style given in Fig. 5(a). Each adjacent pair of ports can be considered to be connected by a transmission line of electrical length θ and characteristic admittance Y. This example provides a good illustration of the matching principles developed in the previous section. For this special case, it may be shown that (17) reduces to

$$G_e = Y_{12}\frac{\sqrt{3}}{2} \tag{20}$$

$$B_e = Y_{11} + Y_{12} + \frac{Y_{12}}{2} \tag{21}$$

where Y_{11}, Y_{22}, and Y_{12} are the admittance matrix entries of the two-port network connecting each of the five-ports to its neighbors. In the case of a transmission line of electrical length θ and characteristic admittance Y

$$Y_{11} = Y_{22} = -Y\cot\theta, \quad Y_{12} = Y/\sin\theta \tag{22}$$

so that

$$Y_{eq} = \frac{\sqrt{3}\,Y}{2\sin\theta} + jY\left(\frac{1}{2\sin\theta} - 2\cot\theta\right). \tag{23}$$

The circuit form for this admittance is given in Fig. 6(a). There are two parameters which may be adjusted to obtain a match at a particular frequency, namely the electrical length θ and the admittance level Y. We must have $G_e = 1$ and $B_e = 0$ for a match. The susceptance will be zero if the imaginary part of (23) is zero. This will be the case if $\theta = 2\pi l/\lambda = \cos^{-1}(1/4) = 75.5°$. The real part of (23) will be 1 if

$$Y = \frac{2\sin\theta}{\sqrt{3}} = \sqrt{5/2} \simeq 1.12.$$

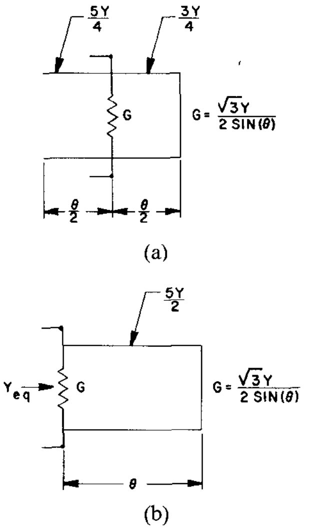


Fig. 6. The circuit form of the equivalent admittance of the ring style five-ports given in Fig. 5(a) and (b). The circuit form given in Fig. 6(b) is particularly simple being a nearly constant conductance $\sqrt{3}\,Y/(2\sin\theta)$ shunted by a short-circuited stub of electrical length θ and admittance level $5Y/2$.

By including a shorted section of transmission line $\theta/2$ long with characteristic admittance $Y/2$ at each port, as in Fig. 5(b), the equivalent admittance takes on the following simple form:

$$Y_{\text{eq}} = \frac{\sqrt{3}}{2}Y\left\{\frac{1}{\sin\theta} - j\frac{5}{\sqrt{3}}\cot\theta\right\} \tag{24}$$

as represented by the simple circuit given in Fig. 6(b). It is of the same form as that of the two-branch coupler with 2.1-dB coupling and the theory developed to broadband this component with external matching networks can be applied to this case [7]. The intrinsic bandwidth will be about the same as that of the two-branch hybrid or 10 percent for a return loss of 20 dB.

VI. Matched Stripline Junction Five-Port

The analogy with the three-port junction circulator suggests the possibility of trying to construct a matched stripline junction five-port of the form given in Fig. 7. This junction five-port might be expected to be less reactive then the ring five-port, and, as a result, to have greater intrinsic bandwidth. Indeed, this has been found to be the case. Several stripline junction five-ports of this form have now been built with greater than an octave bandwidth.

The analysis of this device proceeds in a manner similar to that for the junction circulator. From (17), (18), and (19), the equivalent admittance can be evaluated if the eigenadmittances Y_1, Y_2, and Y_3 are known. Moreover, theoretical expressions for these are available in the literature in terms of Bessel function series [8]. These expressions assume that the tangential magnet field H_θ is constant across the striplines where they connect to the junction and zero elsewhere. In practice, calculations proceed by replacing the physical conductor width by an equivalent parallel plate conductor width which yields the same characteristic impedance for the striplines. The field is then assumed to be constant across this equivalent width and zero elsewhere. We have found the surprising result that for an equivalent coupling angle $\psi_e \simeq 40°$ the theoretical conductance will be a constant close to one shunted by a small susceptance over about an octave bandwidth. Two experimental stripline models which cover the bands 5–12.5 GHZ and 8–18 GHZ, respectively, confirm this conclusion. The dielectric in both cases was RT-duroid with $\epsilon = 2.22$ and the ground plane spacing was 3.2 mm. The disk diameters d were 17.3 mm and 11.4 mm, respectively. In the case of the 5–12.5 GHZ model $l = 2.7$ mm and W = 4.5 mm, whereas the 8–18-GHZ model was direct coupled ($l = 0$) and required no susceptive matching at the junction. The later can serve to design direct coupled matched five-port junctions at any frequency by scaling. First, the diameter must be adjusted to correspond to the frequency band of interest. Then, the ground plane spacing must be adjusted to give an equivalent coupling angle $\psi_e \simeq 40°$ for 50-Ω connecting striplines.

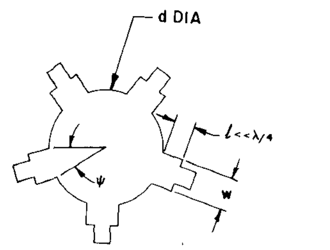


Fig. 7. Experimental stripline five-port junction with short low-impedance line sections for matching.

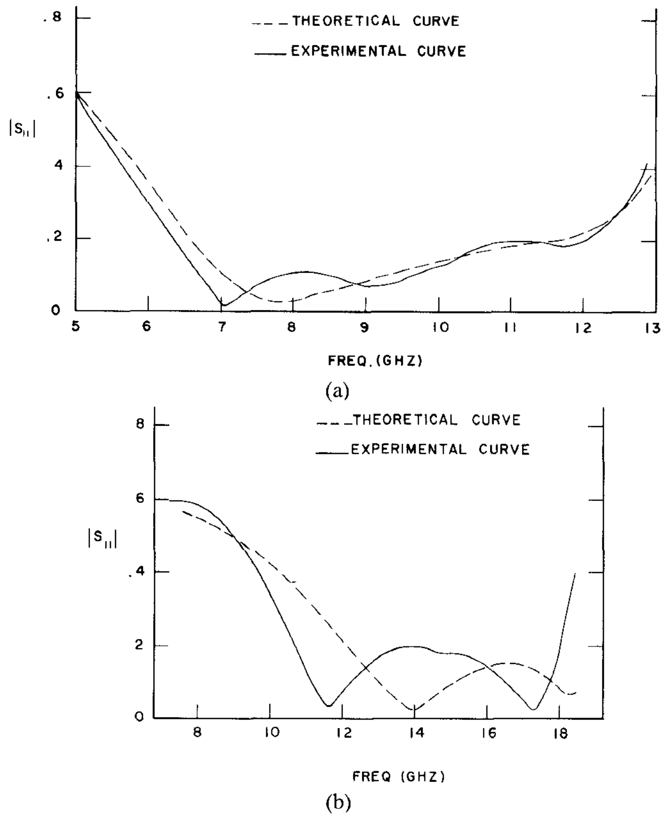


Fig. 8. Theoretical and experimental values of $|S_{11}|$ for two stripline five-port junctions of the type given in Fig. 7 ($\epsilon = 2.22$, gps. = 3.2 mm). (a) $d = 17.3$ mm, $l = 2.7$ mm, $W = 4.5$ mm. (b) $d = 11.4$ mm, $l = 0$.

In Fig. 8, a comparison is given between the experimental and theoretical values of $|S_{11}|$ for these two units. In

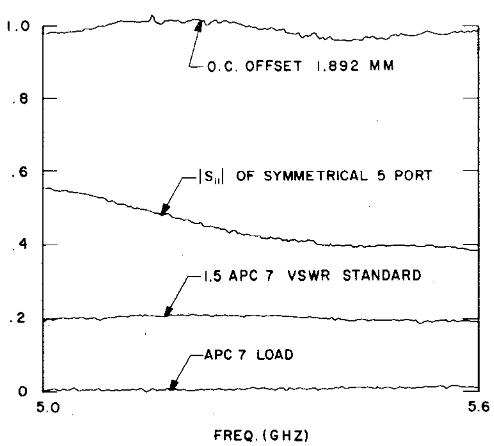


Fig. 9. Measurements of some standard mismatches using the stripline five-port junction of Fig. 8(a) in the six-port configuration of Fig. 2. The sloping curve is the measurement of $|S_{11}|$ of a second identical five-port junction terminated in loads and illustrates the measurement of an actual component.

both cases, two frequencies of best match are predicted and observed experimentally. In practice, it has been found that the units, when properly calibrated, may be used to make accurate measurements of Γ even if there is a substantial mismatch at the input ports of the five-port [3]. This is consistent with the general philosophy of calibrating six-ports to correct for substantial departures from ideal performance. Data on the measurement of standard mismatches which confirm this conclusion has been given in preliminary reports on this work. At the low frequency end, good measurements can be made even with $|S_{11}| \simeq 0.5$. Fig. 9 gives some measurements of some standards from 5–5.6 GHZ where $|S_{11}| \simeq 0.5$ and which substantiate this assertion. A measured curve is also given of $|S_{11}|$ of a second identical 5–12.5-GHZ stripline five-port terminated in loads as an example of the measurement of an actual component.

In the measurement procedure, the reference detector is connected to the reference plug-in of a PMI 1038 D14 mainframe with the IEEE bus. The channel B plug-in is replaced with a switch box controlled by a PET 2001 personal computer and which switches the three remaining detectors of the six-port in turn to the channel A plug-in. The PET 2001 controller has been machine language programmed to enable a very rapid calculation of the reflection coefficient components. A curve such as those given in Fig. 9 and based upon 300 frequency points appears on the screen of the PMI about 30 sec after the measurement procedure is initiated.

VII. Conclusions

In this paper, several types of matched stripline five-ports for use in making six-port measurements have been described. Although the matched five-port ring style is the easiest to design theoretically, the junction five-port is quite broadband for an equivalent coupling angle $\psi_e \simeq 40°$. There are two frequencies of nearly perfect match. It has been demonstrated experimentally, however, that excellent measurements of Γ can be made over a bandwidth greater than an octave and at frequencies where there is a substantial mismatch so that ideal performance cannot be expected. This is consistent with the general spirit of six-port measurements.

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